Recursive Bias-Correction Method for Identification of Piecewise Affine Output-Error Models

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Outline

Problem: PWA-OE identification

Recursive PWA-OE identification algorithm

Numerical Example

Conclusions

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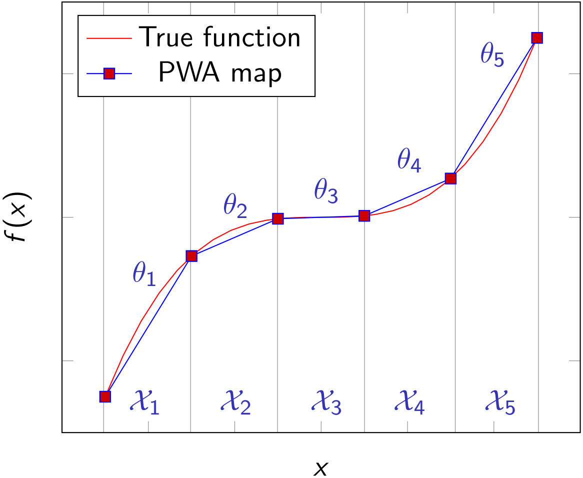
PWA models



$$f(x) = \begin{cases} \theta_1 \, x & \text{if } x \in \mathcal{X}_1 \\ \vdots & \\ \theta_s \, x & \text{if } x \in \mathcal{X}_s \end{cases}$$

Problem: PWA-OE identification

PWA Output-Error data-generating system:

$$y_{\mathrm{o}}(k) = f(x_{\mathrm{o}}(k));$$
$$y(k) = y_{\mathrm{o}}(k) + e_{\mathrm{o}}(k); \quad e_{\mathrm{o}}(k) \sim \mathcal{N}(0, \sigma_e^2)$$

$$f(x_{\mathrm{o}}) = \begin{cases} (\theta_1^{\mathrm{o}})^\top \begin{bmatrix} x_{\mathrm{o}} \\ 1 \end{bmatrix} & \text{if } x_{\mathrm{o}} \in \mathcal{X}_1 \\ \vdots & \\ (\theta_s^{\mathrm{o}})^\top \begin{bmatrix} x_{\mathrm{o}} \\ 1 \end{bmatrix} & \text{if } x_{\mathrm{o}} \in \mathcal{X}_s \end{cases}$$

$x_{\mathrm{o}}(k)$ is the noise-free regressor

$$x_{\mathrm{o}}(k) = [y_{\mathrm{o}}(k-1) \cdots y_{\mathrm{o}}(k-n_a)\ u(k-1) \cdots u(k-n_b)]^\top$$

$\{\mathcal{X}_i\}_{i=1}^s$ are the polyhedral partition of the regressor space.

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Model structure:

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Assumptions:

- model structure n_a, n_b, s is fixed
- y_{o} and u are bounded and statistically independent of e_{o}.
- all s modes are sufficiently excited.

Goal:

Given $\{u(k), y(k)\}_{k=1}^{N}$,

- compute consistent estimates of $\{\theta_i^{\mathrm{o}}\}_{i=1}^{s}$
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Given the partition $\{\mathcal{X}_i\}_{i=1}^{s}$, LS estimate of the i-th model is

$$\theta_i^{\mathrm{LS}} = \left(\frac{\mathbb{X}_i^\top \mathbb{X}_i}{N_i}\right)^{-1} \frac{\mathbb{X}_i^\top \mathbb{Y}_i}{N_i}$$

$\mathbb{X}_i, \mathbb{Y}_i$ measured noisy regressors, output of the i-th model.

- It can be proved that LS estimate θ_i^{LS} is biased!
- Asymptotic bias is caused due to OE structure:

$$\lim_{N_i \to \infty} \theta_i^{\mathrm{LS}} = \theta_i^{\mathrm{o}} + \lim_{N_i \to \infty} \underbrace{B_\Delta(\theta_i^{\mathrm{o}}, \mathbb{X}_{i,\mathrm{o}})}_{\text{bias}}$$

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Bias-correction: Quantify and remove the bias from LS estimates

$$\theta_i^{\mathrm{o}} = \theta_i^{\mathrm{LS}} - \underbrace{B_\Delta(\theta_i^{\mathrm{o}}, \mathbb{X}_{i,\mathrm{o}})}_{\text{bias}}$$

However, $B_\Delta(\theta_i^{\mathrm{o}}, \mathbb{X}_{i,\mathrm{o}})$ depends on $\theta_i^{\mathrm{o}}, \mathbb{X}_{i,\mathrm{o}}$ and can not be computed.

- Key idea: We define the bias-corrected estimate θ_i^{BC}

$$\theta_i^{\mathrm{BC}} = \theta_i^{\mathrm{LS}} - B_\Delta(\theta_i^{\mathrm{BC}}, \Psi i)$$

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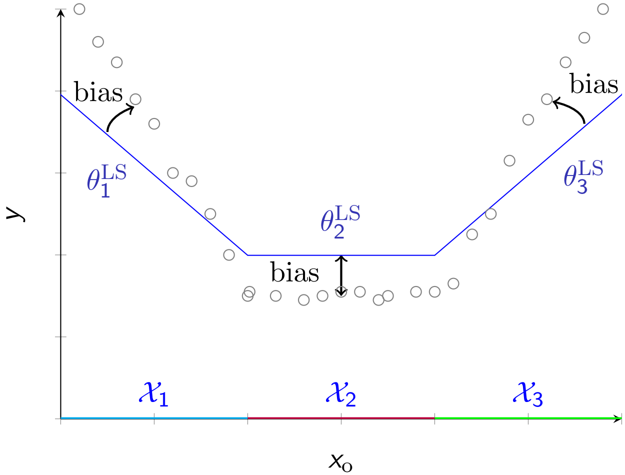
Recursive bias-corrected estimate θ_i^{BC}

$$\theta_i^{\mathrm{BC}}(k) = \theta_i^{\mathrm{LS}}(k) + \hat{\sigma}_e^2(k-1)\kappa_i P_i(k) J \theta_i^{\mathrm{BC}}(\tau).$$

with

- $\theta_i^{\mathrm{LS}}(k)$ parameter update with RLS algorithm
- $P_i(k)$ covariance update via RLS algorithm.
- $\hat{\sigma}_e^2(k-1)$ noise variance estimate.

Bias-corrected least squares estimates



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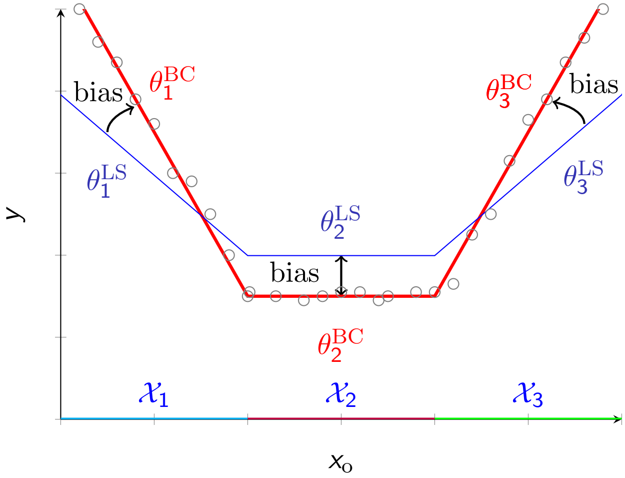

Clustering the regressors

The active mode $\sigma(k)$ is defined as

$$\sigma(k) = i \quad \Leftrightarrow \quad x_{\mathrm{o}}(k) \in \mathcal{X}_i,$$

The proposed clustering criterion to estimate $\sigma(k)$

$$\underbrace{\sigma(k)}_{\text{active mode}} \leftarrow \arg \min_{i=1,\ldots,s} \lambda e_i^2(k) + \|\hat{x}_{\mathrm{o}}(k) - c_i\|_2^2;$$

- $e_i(k) = y(k) - (\theta_i^{\mathrm{BC}})^\top \begin{bmatrix} \hat{x}_{\mathrm{o}}(k) \\ 1 \end{bmatrix}$: prediction error.
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Recursive parameter estimation and clustering

1. **for** $k = 1, \ldots, N$ **do**
   1.1 **active mode** $\sigma(k) \leftarrow \arg\min_{i=1,\ldots,s} \ \lambda e_i^2(k) + \|\hat{x}_{\rm o}(k) - c_i\|_2^2$;
   1.2 **update** model parameters

$$\theta^{\rm BC}_{\sigma(k)} \leftarrow \theta^{\rm LS}_{\sigma(k)} + \hat{\sigma}_e^2(k-1) N_{\sigma(k)} P_{\sigma(k)} J \theta^{\rm BC}_{\sigma(k)};$$

   1.3 **update** variance $\hat{\sigma}_e^2(k) = \frac{k-1}{k}\hat{\sigma}_e^2(k-1) + \frac{1}{k}\left(y(k) - \hat{y}_{\rm o}(k)\right)^2$;
   1.4 **update** centroid $c_{\sigma(k)} \leftarrow c_{\sigma(k)} + \frac{1}{N_{\sigma(k)}}(\hat{x}_{\rm o}(k) - c_{\sigma(k)})$;
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PWA-OE data-generating system

$$y_{\rm o}(k) = \begin{cases} [\,-0.4 \;\; 1 \;\; 1.5\,]\, x_{\rm o}(k), & \text{if } 4y_{\rm o}(k-1) - u(k-1) + 10 < 0, \\ [\,0.5 \;\; -1 \;\; -1.5\,] x_{\rm o}(k), & \text{if } 4y_{\rm o}(k-1) - u(k-1) + 10 < 0, \\ & \&5y_{\rm o}(k-1) + u(k-1) - 6 \leq 0, \\ [\,-0.3 \;\; 0.5 \;\; -1.7\,]\, x_{\rm o}(k), & \text{if } 5y_{\rm o}(k-1) + u(k-1) - 6 > 0, \end{cases}$$

$$y(k) = y_{\rm o}(k) + e_{\rm o}(k)$$

with $N = 5000$ training samples and SNR $= 11.7$ dB.

- Identification of PWA model with $s = 3$, $n_a = n_b = 1$
- Recursive algorithm is run with $\lambda = 1.8$ and different initial guesses for noise variance $\sigma_e^2(0)$.

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Norm of the parameter estimation error LS *vs* BC estimates.

Mode	$\left\|\theta^{\mathrm{o}} - \theta^{\mathrm{BC}}\right\|$	$\left\|\theta^{\mathrm{o}} - \theta^{\mathrm{LS}}\right\|$
$s = 1$	0.0532	0.7500
$s = 2$	0.0223	0.2214
$s = 3$	0.1161	0.1573

Mode fit index $\mathrm{MF} = \left(\frac{1}{N}\sum_{k=1}^{N} \mathbb{I}(\sigma(k) = \sigma^{\star}(k))\right)$

	BC	LS
MF	0.81%	0.73%

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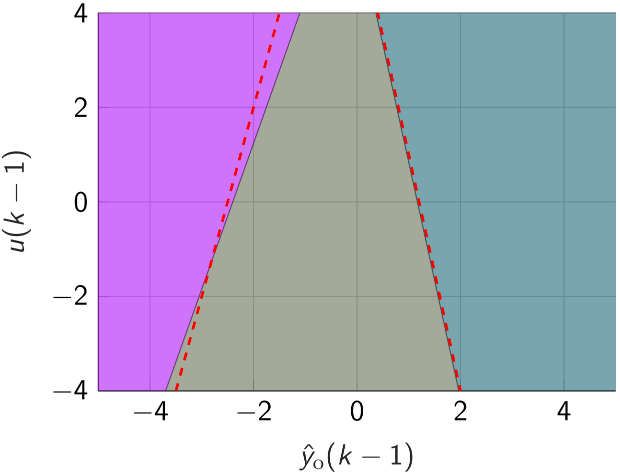

True (dashed red lines) *vs* estimated polyhedral partition.

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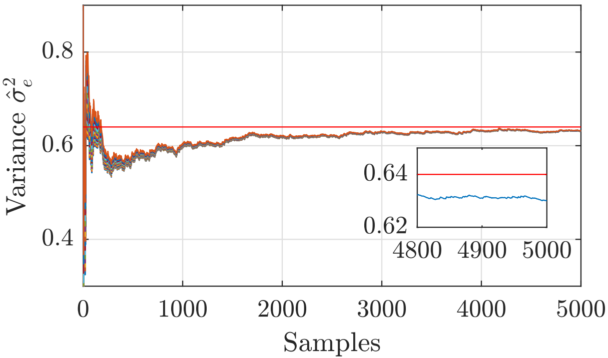

True (solid red) *vs* estimated variance

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Conclusions

- A recursive algorithm for identification of PWA-OE models is proposed.
- Bias-correction scheme is combined with a clustering algorithm for recursive identification.
- The parameters are consistent under suitable assumptions.
- Future work will be focused on data-driven model structure selection.

Thank You